Full Class Discussion

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37. **Rates vs Totals** - Students compare points on an hours/pay Cartesian space. The main challenge lies in recognizing that, although one student earned more, the other student was paid better, that is, at a higher rate of pay. They must indicate the difference in pay and the differences in amount worked.
38. **Recipes that Exchange** - The lesson focuses on a function that multiplies input by two but also changes the ingredient to another type of ingredient.
39. **Rules and Formulas** - Students are given a rule and a data table supposedly generated according to the rule. Students evaluate whether: (1) the proper rule has been applied and (2) the result is correct.
40. **Starting With A Rule** - Students focus on whether given outputs are consistent with a given rule.
41. **Symbols** - Discussion about what symbols are; writing messages or "stories" with symbols; interpreting symbols.
42. **Three Heights** - In this class we will explore: (a) How the children deal with comparisons, (b) How they draw inferences from comparisons, and (c) How they represent comparisons between three unknown amounts.
43. **Time and Time Lines** - Students will discuss and learn about points and intervals on time lines of various sorts.
44. **Times Two** - The lesson focuses on a function that multiplies the input by two. New notations are introduced.

Fourth Grade Lessons

1. **Cartesian Candy Bars I** - We compare ratios of various ordered pairs in a Cartesian grid. The initial discussion concerns the space as a whole; the task will focus on selected points and on the ratio of the dependent variable to the independent variable.
2. **Cartesian Candy Bars II** - Children work on sharing different amounts of candy bars among different numbers of people. They compare ratios (candy bars per person) and plot points in a Cartesian grid.
3. **Comparing Functions** - This lesson is split into two days. In the first class, the students will analyze eight basic graph shapes and will represent and solve a verbal problem involving the choice between two functions. In the second one they will be asked to choose, among the eight basic graph shapes, the ones that matches specific situations.
4. **Consistency** - Children choose pairs of numbers that maintain the relationship of 1 to 3 that is given in a statement, and they explain why they believe the relationship is maintained.
5. **Equations and Inequalities** - Students will work with equations and inequalities, first with simple ones and later with comparisons of two functions. The Wallet Problem, introduced in a previous lesson, will provide the background context.
6. **Evaluation Problem** - Students will be given a problem that asks about the amount of money each person has, based on the amount in a piggy bank. They will be given one graph and asked to draw the second graph.
7. **Graphing A Story** - A trip is described in miles, hours, and miles/hr. Students produce a graph from the description. They then produce a table from the graph and answer questions about the trip.
8. **Graphing Halves and Doubles** - Children work on a problem about distance and time and compare two rates: half a meter per second and two meters per second.
9. **Graphing Thirds and Triples** - Children work on a problem about distance and time and compare two rates: one third of a meter per second and three meters per second.
10. **Intervals** - Students reason about graphs showing growth over time. They compare heights of children and heights of two animals at different time intervals.
11. **Multiplicative Candy Boxes I** - This class centers on the possible amounts of candies two children, Juan and Marcia, have. Juan has a box of candy and Marcia has twice as much candy. What are the possible amounts of candies they might have?
12. **Multiplicative Candy Boxes II** - This class is a continuation of the Multiplicative Candy Boxes I lesson. It centers on the possible amounts of candies two children, Juan and Marcia, have. Juan has a box of candy and Marcia has twice as much candy. What are the possible amounts of candies they might have?
13. **Running Race I** - Compare a race between two students: one who runs at a constant pace, the other who tires out as the race proceeds.
14. **Running Race II** - Compare a race between two students: one who runs at a constant pace and one who changes pace as the race proceeds.
15. **Swimming Pools I** - Compare how two swimming pools fill up with water over several hours.
16. **Swimming Pools II** - Students will examine the rate of pools filling over several hours.
17. **The Better Paying Job I** - Children work on a problem about rate of pay per hour of work. They compare ratios (dollars earned per hour of work) and discuss and plot points in a Cartesian plane.
18. **The Better Paying Job II** - Children work on a problem about rate of pay per hour of work. They compare ratios (dollars earned per hour of work) and discuss and plot points in a Cartesian plane.
19. **Three Heights Review** - In this class we will explore: (a) How children deal with comparisons, (b) How they draw inferences from comparisons, and (c) How they represent comparisons between three unknown amounts.
20. **Three to One** - Children discuss and produce verbal and mathematical statements on the proportion, S:L :: 1:3, that is, on the function f(x) = 3x and on its inverse f $^{-1}$(x) = 1/3 x
21. **Two Phone Plans I** - Students compare two phone plans, one of which has a lower rate, but a monthly basic charge; the other has a higher rate but no basic charge.
22. **Two Phone Plans II** - Students will work on the comparison between two phone plans (also used in the lesson "Two Phone Plans I"), one of which has a lower rate, but a monthly basic charge, the other has a higher rate but no basic charge.
23. **Varying Speed** - Children are asked to tell a story about a trip depicted through a graph that has varying slopes/speeds.
24. **Varying Velocity** - Children are asked to tell a story about a trip depicted through a graph that has varying slopes/velocities.
25. **Wallet Problem I** - Students compare the amounts of money two students have. The amounts are described relationally but not through precise dollar amounts.
26. **Wallet Problem II** - Students will be given a wallet problem. They will be asked to compare the amounts of money two students have. The amounts are described relationally but not through precise dollar amounts.
27. **Wallet Problem III** - Students will continue working with the wallet problem. They will be shown a graph for Mike's amounts and asked to (a) determine whether it represents Robin's or Mike's money and (b) to predict where the line for Mike would fall. Later they will plot Mike's amounts and will discuss why the lines cross.

Fifth Grade Lessons

1. **Arcade** - Students are told a story about two children, each of whom has a certain amount of money, but only one of whom has an amount known to us. After a series of events they happen to end up with the same amount of money.
2. **Basic Function Shapes** - In this lesson, the students will (a) discuss, represent, and solve a verbal problem involving the choice between two functions; (b) choose, among 8 basic graphs (7 distinct shapes), the one that matches specific situations; and (c) write stories to match a specific graph shape.
3. **Elapsed Time** - A variant of the train crash problem is used to address questions about elapsed time. The task is to determine where a train is, given a certain time.

4. **Enacting and Solving Equations** - Students enact and discuss a situation where two children have amounts of candies. Some of the candies are visible, others are inside opaque tubes or boxes. After considering multiple possibilities they are told that the children have the same amount of candies. The situation corresponds to the equation $3x + y + 6 = x + y + 20$, where x is the amount of candies per tube and y is the amount of candies per box. Students will be asked to discuss and to represent the situation, to solve the equation that corresponds to the situation, and to solve other written equations with similar structure.
5. **Equations and Graphs** - Students will further compare two linear functions in the context of evaluating two plans for shoveling snow. One plan has two parts: a basic charge plus a charge based on the number of square meters cleared. The other plan has no basic charge; it only charges according to the number of square meters cleared. However the per-meter charge is higher than in the other plan. Students are asked to determine the circumstances in which the bill from each plan would be the same. They then examine the graph of the two functions and discuss how equations and inequalities relate to the graph.
6. **Equations in Groups** - Students first discuss equality situations and how equal changes on both sides of the equality do not change the equality or the solution to the equation. In a second activity, A pair of students begins with a solved equation (e.g. N = 4) and passes the equation to their neighbor; the neighbor operates equally on each side of the equation and passes the equations to the following neighbor. They continue this process until the series of equations return to the first two students who, then, check whether the solution still holds. They also check the logic and correctness of their colleagues operations on the initial equation.
7. **Equations in Groups II** - A student (or a pair of students) begins with a solved equation (e.g. N = 4) and pass(es) the equation to neighbor (or pair of neighbors); the neighbor(s) operate(s) equally on each side of the equation. And so on, around the table. There should be at least three students or pair of students at each table. When the series of equations returns to the first students, each student (or pair of students) check whether the solution still holds for the solution they had proposed at the beginning. They also check the logic and correctness of the changes implemented by their classmates.
8. **Fifth Grade Assessment I Review** - This lesson will focus on reviewing the recent in-class assessment, on writing equations for word problems, and on solving equations.
9. **Phone Plans** - Students will compare two linear functions in the context of evaluating phone plans. One plan has two parts: a basic charge plus a charge based upon the number of minutes used. The other plan has no basic charge; it only charges according to the minutes used. However the per-minute charge is higher than in the other plan. Students are asked to determine the circumstances in which the monthly bill from each plan would be the same. They then examine the graph of the two functions and discuss how equations and inequalities relate to the graph.
10. **Review on Graphs and Equations** - In this lesson, the students will solve individually or in small groups the set of problems. For each problem, the teacher will lead a discussion based on the students' work (the teacher should identify strong and weak points in the students' work). The class is organized around four main problems. Within each problem students will answer different questions.
11. **Solving Equations I** - Students will be asked to use the syntactic rules of algebra to solve equations with variables on both sides of the equals sign.
12. **Solving Equations II** - Students will be asked to represent and solve verbal problems requiring algebra and to use the syntactic rules of algebra to solve equations with variables on both sides of the equals sign.
13. **Solving Equations with One Variable** - Students work on a story about two children who each have a certain amount of money. The amount of one of the children is known but the other is not. After a sequence of transformations they end with the same amount of money. Students will be led to solve for the starting value by relating the equation to the events in the story. After that, they will be asked to solve another similar problem.
14. **Train Crash** - Students will compare two linear functions represented in a graph. They reason about the problem using (a) the word problem and two diagrams; (b) a graph of position vs. time; (c) a table of values (d) making expressions for each position function; and (e) solving the equation algebraically.
15. **Varying Rates of Change** - Students will compare three functions, two of which are nonlinear, that tell the story of three cousins who all save $1,000 in one year. One saves a lot the first day and less and less each day as time goes on; one saves very little the first day and more and more each day throughout the year; the last cousin saves the same amount each day. Students will be asked to predict the shape of the graph for each function and, later, to look at and describe graphs of all three cousins' savings.
16. **Wallet Review Problem** - This activity is a review of the Wallet Problem done in fourth grade. It is intended to introduce new students to some of the concepts we have covered and to refresh the memories of our old students. Students compare the amounts of money two students have. The amounts are described relationally but not through specific dollar amounts.

Middle School Lessons

1. **Biggest Output** - Students will decide on what linear and quadratic functions will result in the greatest output, starting from an algebraic expression, and using tables and graphs to help them make these decisions.
2. **Box Extremum** - Students will start by finding average rates of change for a non-linear function over increments of the independent variable. The size of the increments will decrease to introduce the idea of using tangent lines to find instantaneous rates of change of linear and non-linear functions. Students will see what a tangent looks like at the extrema of a graph. Students will then create a box that maximizes the volume and see how determining the extrema of a graph can help to find the maximum volume.
3. **Box of Clay Activity** - Students will compare two cubic functions based on the context of the volumes of a box of clay.
4. **Can We Predict Differences?** - Students will predict, produce, and compare linear and non-linear function graphs used to represent the number of punches on a balloon.
5. **Contrasting Equations** - Students write equations for three graphs and examine their slopes by comparing and contrasting the graphs. Students also look at the same functions graphed on differently scaled coordinate planes.
6. **Coupon Activity** - Students will create graphs, tables and equations to explain their stories and look at how a graph changes depending on the y-intercept.
7. **Curves in a Cubic** - Students will explore different kinds of cubic functions through graphs and tables.
8. **Function Challenges - 20 Questions** - Students will compete in a game to generate equations for functions that meet certain criteria, as given by the instructor.
9. **It Depends** - Students will think about how we can show a dependent relationship between two quantities, using a variety of representations.
10. **Jason's Tree House** - Students will extract data from a story and use tables and graphs to answers questions about proposed scenarios.
11. **Lotto Winnings** - Students will generate a graph for a nonlinear function, point by point, in order to realize that there are different types of functions that they might not know about yet.
12. **Playground Construction** - Students will create a quadratic equation based on the context of building a playground referring to surface, fencing, and equipment needed, to create an equation of $y = ax^2 + bx + c$ form.
13. **Race Car Activity** - Students will look at four different graphs to determine which two describe the scenario proposed by the teacher displaying parallel lines and the correct y-intercepts.
14. **Same and Different** - Students will compare graphs of linear functions, looking for similarities and differences, and will produce algebraic expressions, again identifying what is the same and what is different about each one.
15. **Who Shares My Function? - Linear with All Representations** - Students will work in groups after finding other students who have the same linear function represented by a story, a table, a graph, or an equation. They will attempt to explain and discuss why the different representations refer to the same function.

16. **Who Shares My Function? - Linear with Graphs and Stories** - Students will make groups by finding other students who have the same quadratic or linear function in different representations.
17. **Who Shares My Function? - Linear with Graphs, Tables, and Equations** - Students will make groups by finding other students who have the same linear function, as shown in representations of graphs, tables, or equations. They will then generate a story to go with the function.
18. **Who Shares My Function? - Linear with Negative and Fractional Slope** - Students will find other functions that are the same as theirs, starting from a table, a graph, or an equation. Once they have identified the same function represented in a different way, they will create a story that describes all of the different representations of the same function.
19. **Who Shares My Function? - Quadratics** - Students will make groups by finding other students who have the same quadratic or linear function in different representations.
20. **x2 and x** - Students will look at x squared and x as functions, and for which values of x one function value is greater than the other.