Box Extremum

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Summary	Students will start by finding average rates of change for a non-linear function over increments of the independent variable. The size of the increments will decrease to introduce the idea of using tangent lines to find instantaneous rates of change of linear and non-linear functions. Students will see what a tangent looks like at the extrema of a graph. Students will then create a box that maximizes the volume and see how determining the extrema of a graph can help to find the maximum volume.
Goals	1. Students will understand that a non-linear function has a different "slope", or rate of change, at different values of the independent variable. 2. Students will understand what a tangent is used for on a graph of a non-linear function. 3. Students will begin to discover the uses of a function's extrema.
Materials	Handouts, Scissors, Tape, Popcorn, Computer
Duration	2 hours
Keywords	Contextualized Situations Extrema Full Class Discussion Hands-On Activity Interpretation of Graphs Linear Functions Non-Linear Functions Production of Equations Quadratic Functions Small Group Work Tangent Lines

Activity Plan:

1. Linear Rate (Page 1)

Students will be given a handout about a person who walks at a steady rate (Handout 1). Let the students explore the constant rate through different types of questioning. This linear function should be familiar to students prior to this lesson, but the structure of the questions will lead into the next handout.

Students may tell you how *far* the person walked between 1 minute and 3 minutes (6 feet) or how *far* the person had walked by 2 minutes (7 feet). If this occurs, emphasize that you are looking for the *rate*, in feet / minute.

2. Non-Linear Instantaneous Rates (Page 2)

Give the students Handout 2. Go over this handout as a class. This handout starts off by asking the rates of a person walking by using shrinking time intervals.

For questions 2 and 3, you may wish to draw the lines that represent the average speed over each interval. You can discuss that these lines show a constant rate, but with the same starting times, ending times, and distance traveled as the interval on the non-linear graph.

The lines for each question are given below:

Question	Point on Graph	Secant Line	Rate (ft/min)
2.	Time from 1 to 3 minutes	y=4x - 3	4 ft/min
3.	Time from 1 to 2 minutes	y = 3x - 2	3 ft/min

When the class gets to question 4, solicit some ideas from the students. Then suggest drawing a tangent line. You can discuss how the tangent line arises from making "average rate" (secant) lines over smaller and smaller intervals, until we get to just one point.

The tangent lines for each question are given below:

Question	Point on Graph	Tangent Line	Rate (ft/min)
4.	"1 minute" / (1,1)	y=2x - 1	2 ft/min
5.	"(2,4)" / 2 minutes	y = 4x - 4	4 ft/min
6.	"(0,0)" / 0 minutes	y = 0	0 ft/min
If Necessary	(3,9) / 3 minutes	y = 6x - 9	6 ft/min

On question 6, discuss how the tangent line has a slope of 0, and that in this context, this means the person is standing still. Also, bring up the fact that this is an extremum (in this case, the minimum).

3. Instantaneous Rates (Page 3)

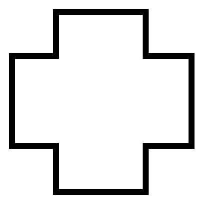
Give the students handout 3 and let them work in pairs.

Review this handout as a class. Make sure to discuss how the rates at (2, -5) and (-2, -5) are the negative opposites of one another.

4. Cube Creation (Page 4)

Give the students Handout 4. Tell them to each create a box that will hold the most popcorn. Make sure they write their name on their box. Students will need scissors and tape.

Be specific that students should make the box by cutting **squares** out of each corner (not other rectangles), and that the square cut from each corner should be the same as all the others. It may help to draw this picture for them:



Go around and put popcorn in everyone's box.

5. Cube Analysis (Page 5)

Allow the students to work individually or in small groups to work on Handout 5.

The table, question 5, should be filled in as a class by soliciting students' choices for *h*.

Make sure to discuss with the class how the volume is calculated with respect to *h*.

h	**Length**	**Width**	**Volume**
h	$30-2h$	$30-2h$	$(30-2h)(30-2h) \times h$
5	20	20	2000
0	30	30	0

6. Cube Simulator

Use the following link to illustrate with a box folding simulator:
http://mathcasts.org/mtwiki/InterA/BoxFolding

In the simulator, change the length to 3 and width to 3. This will correspond to the situation with the handout, 30 units x 30 units, but will differ in the linear dimensions by a scale of 10.

Then, when using the simulator, change only the value of "*h*" so as to show where the values lie on the graph and how it changes the shape of the box. Ask the students which choice of *h* corresponds to the largest volume (i.e. an extremum).

After discussing the simulator pass out the handout on Page 6.

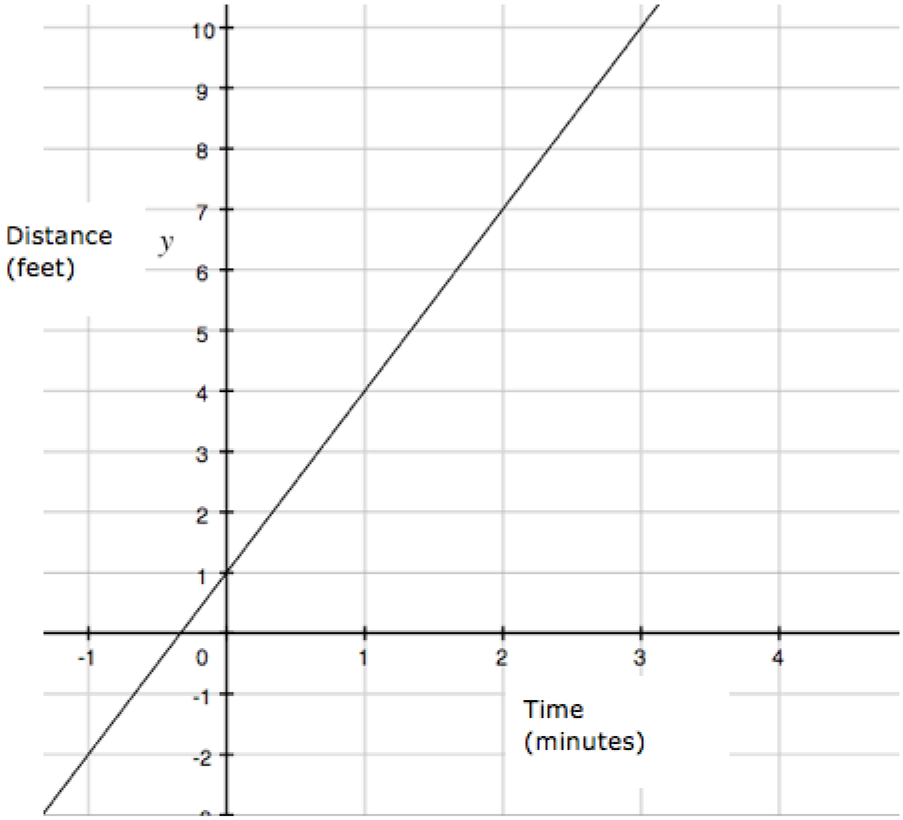
It is important to distinguish the difference between questions 2a and 2b and 4a and 4b. The answers are:

Question	**Answer**
2a	25 units^2 (a 5 unit x 5 unit square)
2b	5 units
4a	0 units^2 AND 225 units^2
4b	0 units AND 15 units

Handout: Walking Rates **(Page 1)**

Name: ___________________________________Date:___________

The graph below represents a person walking.



1. How fast is the person walking during the time from 1 to 3 minutes?

2. How fast is the person walking at 2 minutes?

3. How fast is the person walking at the point (1,4)?

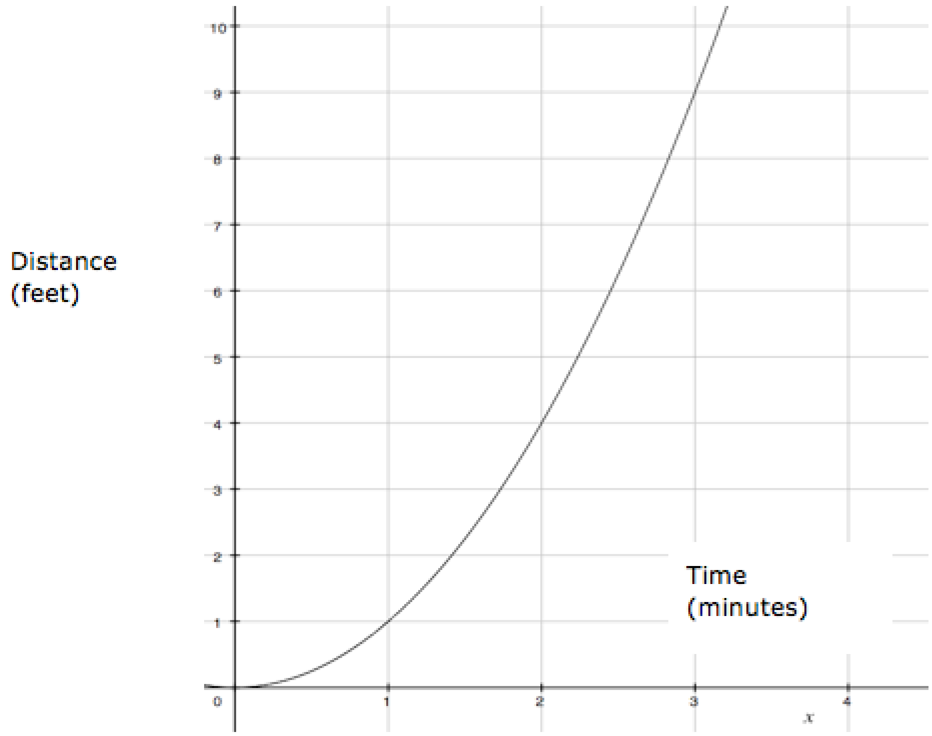
4. What is the equation for the graph? Does the equation show the person's rate?

5. How can you see the person's rate just using the graph?

Handout: Walking Rates **(Page 2)**

Name: ____________________________Date:__________

Below is another graph of a person walking.



1. Do you think the person is walking at a steady rate? Why?

2. How fast is the person walking on average during the time from 1 to 3 minutes?

3. How fast is the person walking on average during the time from 1 to 2 minutes?

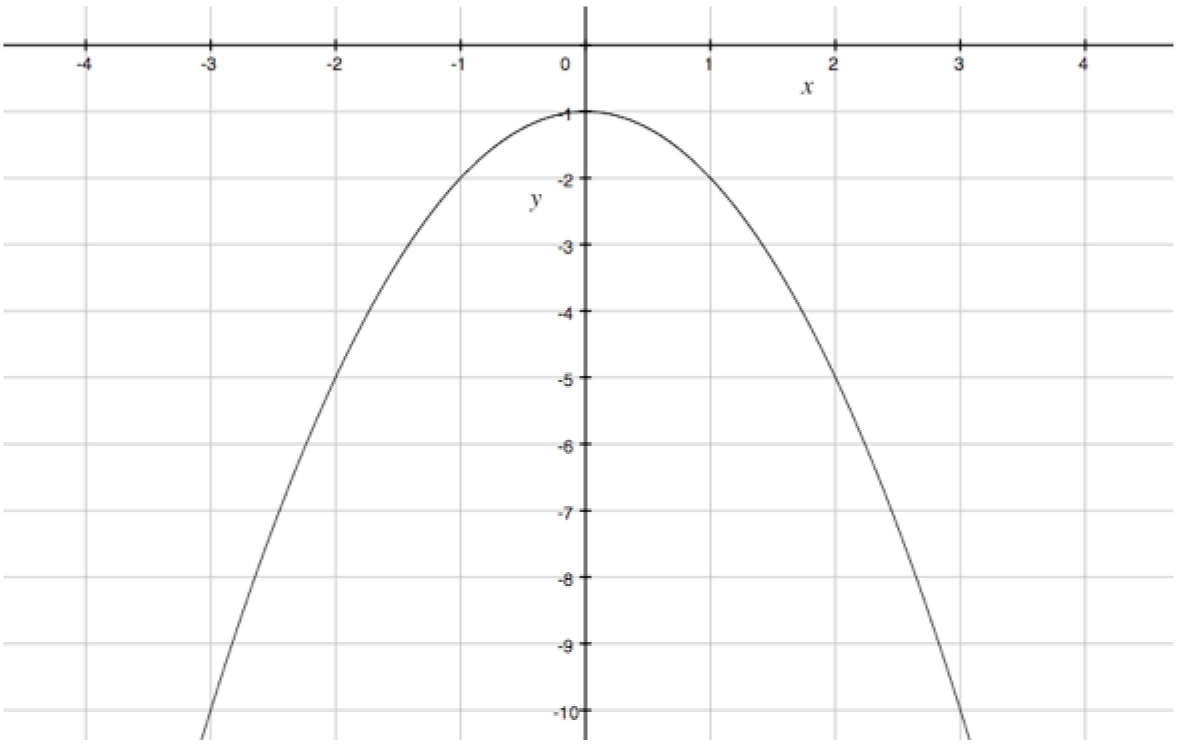
4. Can you tell how fast the person is walking at 1 minute?

5. Can you tell how fast the person is walking at (2,4)?

6. Can you tell how fast the person is walking at (0,0)?

Handout: Instantaneous Rates **(Page 3)**

Name: ___________________________________Date:___________



1. What is the rate of change for the interval x= -3 to x= -1?

2. What is the rate of change for the interval x= -3 to x= -2?

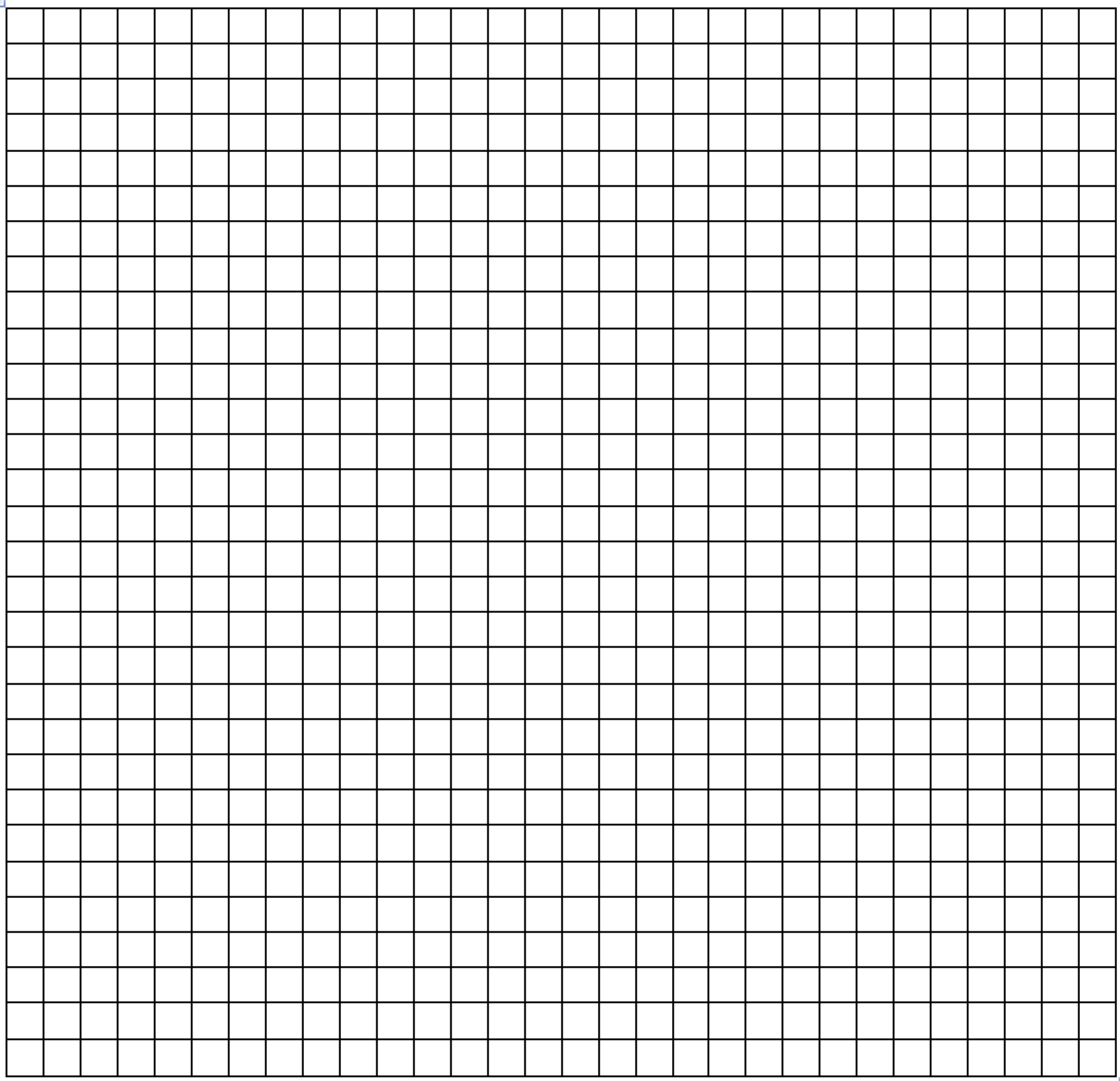
3. What is the rate of change at the point (-2, -5)?

4. What is the rate of change at the point (2, -5)?

5. What is the extremum? How do you know?

Handout: Box Creation **(Page 4)**

Use the graph paper below to create a box **without a top** that will have the largest volume to hold popcorn. You can only **cut out squares** from each corner. (The box below is 30 units x 30 units.)



Handout: Box Folding Analysis (Page 5)

Name: ___________________________________Date:___________

Answer these questions using the box you created before:

1. What is the volume of your box? How much popcorn will your box hold?

2. How do you know the box you created will hold the most possible popcorn?

3. One of your friends disagrees with you. How would you convince them?

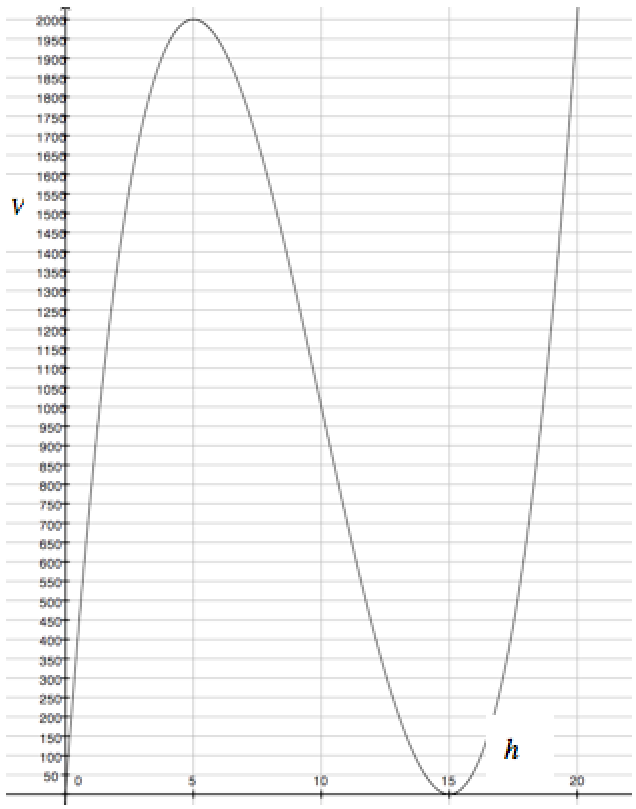
4. How do you create a box that has the minimum volume (or the least amount of popcorn)?

5. Fill in the following table with the given *h* values from other students.

h	**Length**	**Width**	**Volume**

Handout: Box Folding **(Page 6)**

Name: ___________________________________Date:___________



1. Draw the point(s) on the graph where it will show us what to set *h* as so that the volume of the box is the **largest** possible. Label the point(s) as **A**.

2. a) Using the units that were on our box creation, what was the number of squares on each corner we needed to cut out to create the largest volume?

b) So for our box construction the *h* would be: _________ units

3. Draw the point(s) on the graph where it will show us what to set *h* as so that the volume of the box is the **smallest** possible. Label the point(s) as **B**.

4. a) Using the units that were on our box creation, what was the number of squares on each corner we needed to cut out to create the smallest volume?

b) So for our box construction the *h* would be: _________ units