Train Crash

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Summary	Students will compare two linear functions represented in a graph. They reason about the problem using (a) the word problem and two diagrams; (b) a graph of position vs. time; (c) a table of values (d) making expressions for each position function; and (e) solving the equation algebraically.
Goals	1. To relate verbal statements, tables, graphs and algebraic notation. 2. To highlight a virtue of equation-solving-through-manipulation-of-algebraic-expressions (solving for x); namely, precision. The graph, in turn, has its own virtues—it highlights trends and changes over time; it also can represent piecewise functions and other functions that may not correspond to a simple algebraic expression.
Materials	Overheads, Handouts
Terms	Compare/Contrast Equations Contextualized Situation Coordinate Pairs Full Class Discussion Function Representations Inequality Functions Interpretation of Graphs Interpretation of Stories Intersections Linear Functions Production of Algebraic Expressions Production of Equations Production of Tables

	Solving Equations
Hints	Emphasize that an equation can have non-integer solutions and that you can't tell for sure just where a solution lies by looking the graph of an equation.

Activity Plan:

1. Discussing the Word Problem and Diagrams representing the events described in the problem (15 minutes)

Show the Overhead in Page 1 with the problem and the diagrams:

"Two freight trains are racing towards each other along Cannonball Railway. They are out of control. At exactly at 9 a.m.,

- Train A is exactly 25 km north of Greensboro station and traveling north at exactly 50 kilometers per hour.
- Train B is exactly 175 km north of Greensboro station, and racing south, on the same railway track at exactly 70 kilometers per hour.

It looks like the trains are going to crash; we cannot call the conductors.

Your job is to determine exactly when they will crash if the trains keep moving along at the same speed.

You also will need to determine where they are going to crash together. If you know just where they are going to crash, you can at least tell the firemen and ambulances where they should go."

Discuss the overall situation with the children. Ask someone, perhaps several students, to make a drawing of the situation, showing what is known.

Ask the students where the trains would be after one hour (i.e., at 10 a.m.)? [A and B will be 75 km north and 105 km north of Greensboro, respectively]

Where would they be after 2 hours? [A and B will be 125 km north and 35 km north of Greensboro, respectively]

What does this information tell you? [That they either passed each other or they crashed.] We want now to get more precise information about the times and places.

Give them Handout shown in Page 1 and ask them to show with labels the numerical information given in the problem (e.g. the measures, 25 km, 175km, 50 km/h, 70 km/h, and the distance between the two trains at the start and end of their journey.)

1. Handout on Page 2: Comparing the Two Functions in a Graph (15 minutes).

Show the overhead in Page 2 and give students the corresponding handout (also Page 2). Allow them to use the handout 1 in Page 1 as reference. Ask them to think about the following questions:

What does the *y*-axis show? What does the *x*-axis show?

Which train corresponds to which line? How do you know?

Do the trains crash? How do you know?

When do the trains crash?

Where along the railway line will the trains crash? How certain are you about your predictions?

Can you know with precision exactly when the crash will occur? Can you know with precision where it will occur?

What do the lines mean to the right of the crash point?

3. Handout Page 3: Filling in a Table of Values (15 minutes)

Allow students to keep the first two handouts (Page 1 & 2). Give students the handout in Page 3 and have them try to fill in some of the values in the table. More advanced students will try to fill out values on the half-hour and possibly on the quarter hour.

Make sure to discuss the results at length.

4. Handout Page 4: Filling in a Table of Values (15 minutes)

Show the handout in Page 4 and spend some time exploring with the whole class mathematical expressions for representing function a and function b. Make sure to consider the following things:

What units are used for *t*? [hours, *t* means the number of hours passed since 9a.m.]

What units are used for $a(t)$? [km, the number of km north of Greensboro]

What are the functions?

$a(\boldsymbol{t}) = 25 + 50 \times \boldsymbol{t}$ [this should read 25 plus 50 times *t*]

$b(\boldsymbol{t}) = 175 - 70 \times \boldsymbol{t}$ [this should read 175 minus 70 times *t*]

There is a lot to discuss about the difference between: $25 + 50t$ and $175 - 70t$. Why do we add in one of them and in the other we take away? This issue might require a good deal of discussion. It might be useful to have a table for both of them to "see" with numbers how the two trains behave differently.

5. Solving the Equation (Still the handout in Page 4) (15 minutes)

What does it mean to solve the following equation for *t*?

$25 + 50 \times t = 175 - 70 \times t$

What would we have learned if we know the value of *t* that makes the equation true?

6. Discussion: Comparing the Two Functions [whole class] (15 minutes)

Ask a few volunteers to present and discuss their answers.

Go over the solution 150/120 and show how it can be expressed as 5/4 or 1 and 1/4.

What does this mean? [The trains will crash in 1 and 1/4 hours, that is, at 10:15 a.m.]

There are many issues about fractions that will undoubtedly arise. Given them your attention.

How can we find out where the trains will crash?

[e.g. Substitute the elapsed time into $25 + 50 \times t$.

This yields 25 + 50 x 1 and 1/4 or (25 + 50 + 50/4) km]. Remember that this answer is the exact location, north of Greensboro station.

7. Homework (Page 5):

Students are asked to solve the train crash problem on their own. We do not provide a table, graph, or equation template.

Overhead and Handout: A Train Crash (Page 1)

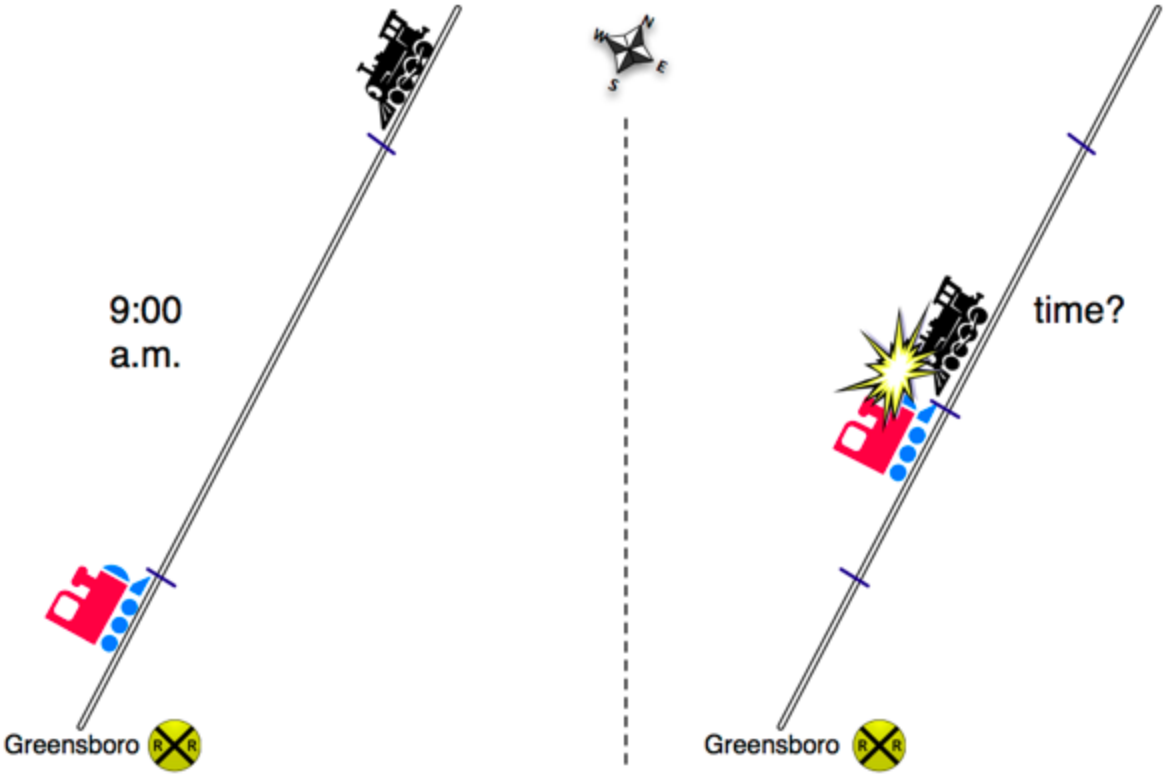
Name: _____ Date: _____

Two freight trains are racing towards each other along Cannonball Railway. They are out of control. At exactly 9 a.m.,

- Train A is exactly 25 km north of Greensboro station and traveling north at exactly 50 kilometers per hour.
- Train B is exactly 175 km north of Greensboro station and racing south, on the same railway track at exactly 70 kilometers per hour.

It looks like the trains are going to crash; we cannot call the conductors.

Place the numbers that you know about in the problem below. Estimate WHEN and WHERE the trains will crash.



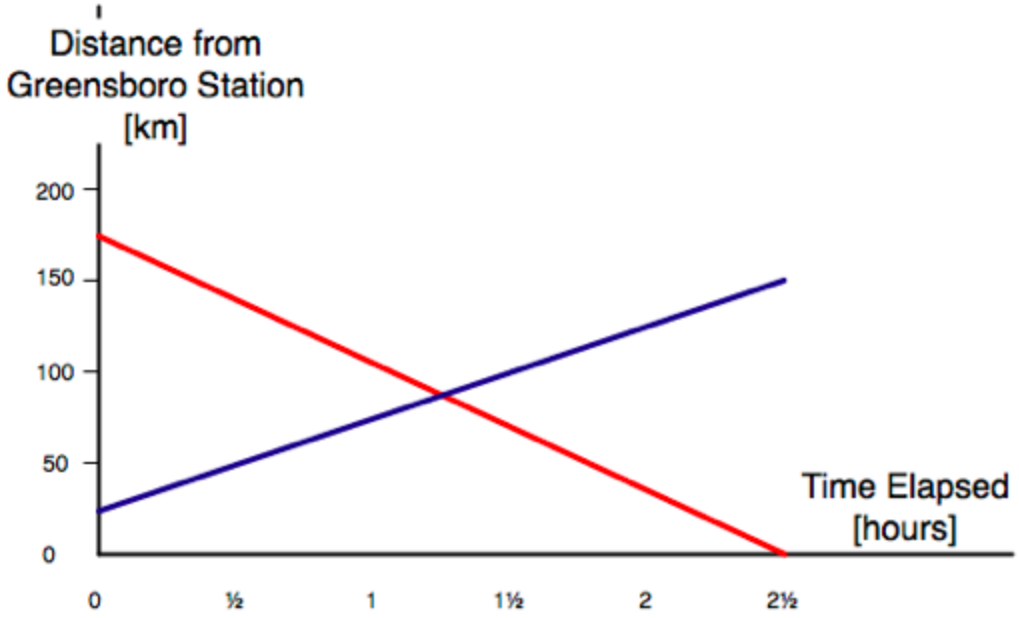
Overhead and Handout: Reasoning from Graphs (Page 2)

Name: _____ Date: _____

Two freight trains are racing towards each other along Cannonball Railway. They are out of control. At exactly 9 a.m.,

- Train A is exactly 25 km north of Greensboro station and traveling north at exactly 50 kilometers per hour.
- Train B is exactly 175 km north of Greensboro station and racing south, on the same railway track at exactly 70 kilometers per hour.

Estimate WHEN and WHERE the trains crash.



My Claim	My evidence
The time when the trains crash is:	
The place where the trains crash is:	

Overhead and Handout: Filling in a Table (Page 3)

Name: _____ Date: _____

Two freight trains are racing towards each other along Cannonball Railway. They are out of control. At exactly 9 a.m.:

- Train A is exactly 25 km north of Greensboro station and traveling north at exactly 50 kilometers per hour.
- Train B is exactly 175 km north of Greensboro station and racing south, on the same railway track at exactly 70 kilometers per hour.

Complete the following table based on the given information.

Time	Time elapsed [hours]	Position Train A: Km north of Greensboro	Position Train B: Km north of Greensboro		Distance between the trains
9:00 a.m.	0 hours	25	175		175-25= 150
	$\frac{1}{4}$ hour				
9:30 a.m.	$\frac{1}{2}$ hour				
10:00 a.m.	1 hour	25 + 50 =	175 -		
10:30 a.m.	$1\frac{1}{2}$ hours				
11:00 a.m.	2 hours				

Overhead and Handout: Solving the Equation (Page 4)

Name: _____ Date: _____

Two freight trains are racing towards each other along Cannonball Railway. They are out of control. Exactly at 9 a.m.

- Train A is just exactly 25 km north of Greensboro station and traveling north at exactly 50 kilometers per hour.
- Train B is exactly 175 km north of Greensboro station and racing south, on the same railway track at exactly 70 kilometers per hour.

It looks like the trains are going to crash; we cannot call the conductors.

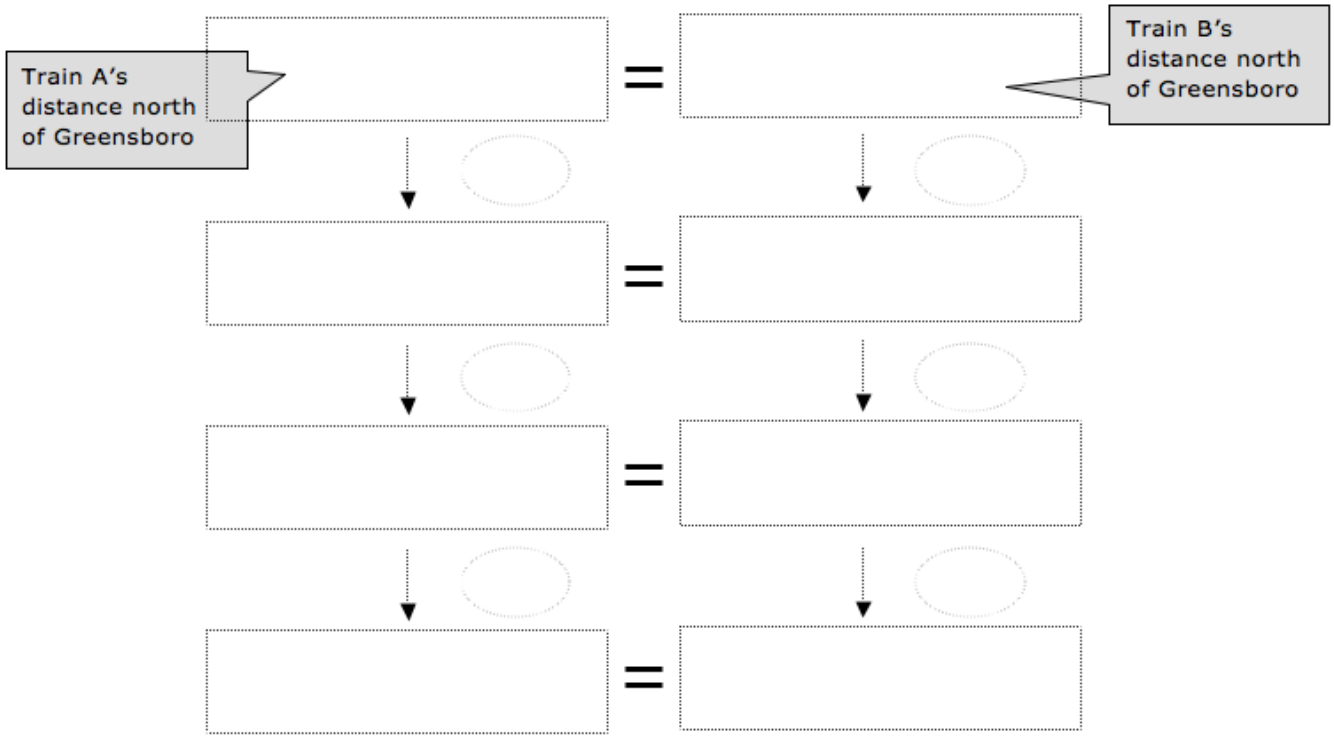
Write expressions for Train A and Train B's distances from Greensboro. The functions are:

Train A's position [km north of Greensboro], $a(t)$ = _____

Train B's position [km north of Greensboro], $b(t)$ = _____

Now write an equation to show that Train A and Train B will meet at some point on the railway line: _____

Solve the equation in order to find out when and where the trains will crash.



Homework: Reviewing Train Problem (Page 5)

Name: _____ Date: _____

Two freight trains are racing towards each other along Cannonball Railway. They are out of control. Exactly at 9 a.m.

- Train A is just exactly 25 km north of Greensboro station and traveling north at exactly 50 kilometers per hour.
- Train B is exactly 175 km north of Greensboro station and racing south, on the same railway track at exactly 70 kilometers per hour.

It looks like the trains are going to crash; we cannot call the conductors.

Write expressions for Train A and Train B's distances from Greensboro. The functions are:
Train A's position [km north of Greensboro], $a(t)$ = _____
Train B's position [km north of Greensboro], $b(t)$ = _____

Show <u>when</u> and <u>where</u> the trains will crash. You can use a picture, a graph, a table, or an equation (that you solve). You can also use words to explain.