

Production of Equations

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Third Grade Lessons

1. **Dots Problem** - We present to the students a problem dealing with a growing pattern over time. To begin, there is one dot. With each passing minute four more dots are drawn around the previous dot(s).
2. **Functions - Earning Money** - The students will create tables and equations from given stories. The functions are additive and multiplicative.

Fourth Grade Lessons

1. **Comparing Strips of Unmeasured Lengths II** - The class is the second of the "Strips of Unmeasured Lengths" series that will focus directly upon the algebraic representation of measurements and their multiplicative relations. Children are asked to compare the lengths of strips, to use algebraic notation to describe the relationships between them, and to demonstrate that the relationships they represent are true.
2. **Comparing Strips of Unmeasured Lengths III** - This is the third lesson in the "Strips of Unmeasured Lengths" series that focuses directly upon the algebraic representation of measurements and their multiplicative relations. We will work with the relationship $B = 3S$, focusing on equations and their verbal descriptions and on true and false equations and statements.
3. **Fourth Grade Assessment II** - This is a written assessment where children will interpret and determine the truth or falsehood of equations and of statements that describe comparisons between quantities.
4. **Piggy Banks** - The whole lesson revolves around a multipart story problem involving changes in two quantities over several days of a week. The initial quantities are equal yet unknown. Then transformations are applied to the quantities. Students are asked to compare the quantities throughout the week even though only their relative relationship can be determined.
5. **Three to One** - Children discuss and produce verbal and mathematical statements on the proportion, $S:L :: 1:3$, that is, on the function $f(x) = 3x$ and on its inverse $f^{-1}(x) = 1/3x$
6. **Wallet Problem III** - Students will continue working with the wallet problem. They will be shown a graph for Mike's amounts and asked to (a) determine whether it represents Robin's or Mike's money and (b) to predict where the line for Mike would fall. Later they will plot Mike's amounts and will discuss why the lines cross.

Fifth Grade Lessons

1. **Arcade** - Students are told a story about two children, each of whom has a certain amount of money, but only one of whom has an amount known to us. After a series of events they happen to end up with the same amount of money.
2. **Enacting and Solving Equations** - Students enact and discuss a situation where two children have amounts of candies. Some of the candies are visible, others are inside opaque tubes or boxes. After considering multiple possibilities they are told that the children have the same amount of candies. The situation corresponds to the equation $3x + y + 6 = x + y + 20$, where x is the amount of candies per tube and y is the amount of candies per box. Students will be asked to discuss and to represent the situation, to solve the equation that corresponds to the situation, and to solve other written equations with similar structure.
3. **Equations and Graphs** - Students will further compare two linear functions in the context of evaluating two plans for shoveling snow. One plan has two parts: a basic charge plus a charge based on the number of square meters cleared. The other plan has no basic charge; it only charges according to the number of square meters cleared. However the per-meter charge is higher than in the other plan. Students are asked to determine the circumstances in which the bill from each plan would be the same. They then examine the graph of the two functions and discuss how equations and inequalities relate to the graph.
4. **Equations in Groups** - Students first discuss equality situations and how equal changes on both sides of the equality do not change the equality or the solution to the equation. In a second activity, A pair of students begins with a solved equation (e.g. $N = 4$) and passes the equation to their neighbor; the neighbor operates equally on each side of the equation and passes the equations to the following neighbor. They continue this process until the series of equations return to the first two students who, then, check whether the solution still holds. They also check the logic and correctness of their colleagues operations on the initial equation.
5. **Equations in Groups II** - A student (or a pair of students) begins with a solved equation (e.g. $N = 4$) and pass(es) the equation to neighbor (or pair of neighbors); the neighbor(s) operate(s) equally on each side of the equation. And so on, around the table. There should be at least three students or pair of students at each table. When the series of equations returns to the first students, each student (or pair of students) check whether the solution still holds for the solution they had proposed at the beginning. They also check the logic and correctness of the changes implemented by their classmates.
6. **Fifth Grade Assessment I** - This assessment will focus on writing equations to solve verbal problems and on solving equations using syntactic rules. It is intended as a diagnostic tool to assist teachers in planning future activities.
7. **Fifth Grade Assessment I Review** - This lesson will focus on reviewing the recent in-class assessment, on writing equations for word problems, and on solving equations.
8. **Fifth Grade Assessment II** - This assessment will focus on writing equations to solve verbal problems and on solving equations using the syntactic rules of algebra.
9. **Phone Plans** - Students will compare two linear functions in the context of evaluating phone plans. One plan has two parts: a basic charge plus a charge based upon the number of minutes used. The other plan has no basic charge; it only charges according to the minutes used. However the per-minute charge is higher than in the other plan. Students are asked to determine the circumstances in which the monthly bill from each plan would be the same. They then examine the graph of the two functions and discuss how equations and inequalities relate to the graph.
10. **Review on Graphs and Equations** - In this lesson, the students will solve individually or in small groups the set of problems. For each problem, the teacher will lead a discussion based on the students' work (the teacher should identify strong and weak points in the students' work). The class is organized around four main problems. Within each problem students will answer different questions.
11. **Solving Equations II** - Students will be asked to represent and solve verbal problems requiring algebra and to use the syntactic rules of algebra to solve equations with variables on both sides of the equals sign.
12. **Solving Equations with One Variable** - Students work on a story about two children who each have a certain amount of money. The amount of one of the children is known but the other is not. After a sequence of transformations they end with the same amount of money. Students will be led to solve for the starting value by relating the equation to the events in the story. After that, they will be asked to solve another similar problem.
13. **Train Crash** - Students will compare two linear functions represented in a graph. They reason about the problem using (a) the word problem and two diagrams; (b) a graph of position vs. time; (c) a table of values (d) making expressions for each position function; and (e) solving the equation algebraically.

Middle School Lessons

1. **Box Extremum** - Students will start by finding average rates of change for a non-linear function over increments of the independent variable. The size of the increments will decrease to introduce the idea of using tangent lines to find instantaneous rates of change of linear and non-linear functions. Students will see what a tangent looks like at the extrema of a graph. Students will then create a box that maximizes the volume and see how determining the extrema of a graph can help to find the maximum volume.
2. **Area of a Square as a Function** - Students will develop a quadratic equation to represent the area of a square.
3. **Box of Clay Activity** - Students will compare two cubic functions based on the context of the volumes of a box of clay.
4. **Can We Predict Differences?** - Students will predict, produce, and compare linear and non-linear function graphs used to represent the number of punches on a balloon.
5. **Candy Experiment** - Students will create their own data to construct a graph and equation of negative and fractional slope.
6. **Contrasting Equations** - Students write equations for three graphs and examine their slopes by comparing and contrasting the graphs. Students also look at the same functions graphed on differently scaled coordinate planes.
7. **Coupon Activity** - Students will create graphs, tables and equations to explain their stories and look at how a graph changes depending on the y-intercept.
8. **Function Challenges - 20 Questions** - Students will compete in a game to generate equations for functions that meet certain criteria, as given by the instructor.
9. **Graphing Equations** - Students will practice moving between graphs and equations of functions, as well as identifying the y-intercept and slope.
10. **Guess My Rule - Linear** - Students will try to determine the equation to match their partner's created graph and work together to correct their own mistakes.
11. **Guess My Rule - Non-Linear** - Students will produce algebraic expressions starting from non-linear graphs produced by other students in the class.
12. **It Depends** - Students will think about how we can show a dependent relationship between two quantities, using a variety of representations.
13. **Jason's Tree House** - Students will extract data from a story and use tables and graphs to answer questions about proposed scenarios.
14. **Lotto Winnings** - Students will generate a graph for a nonlinear function, point by point, in order to realize that there are different types of functions that they might not know about yet.
15. **Playground Construction** - Students will create a quadratic equation based on the context of building a playground referring to surface, fencing, and equipment needed, to create an equation of $y = ax^2 + bx + c$ form.
16. **Race Car Activity** - Students will look at four different graphs to determine which two describe the scenario proposed by the teacher displaying parallel lines and the correct y-intercepts.
17. **Relating Graphs and Equations - Linear and Quadratic Functions** - Students will generate graphs from given equations and equations from given graphs.
18. **Same and Different** - Students will compare graphs of linear functions, looking for similarities and differences, and will produce algebraic expressions, again identifying what is the same and what is different about each one.
19. **Sound Loudness** - Students will examine a non-linear function depicted in a graph and generate the corresponding function table and equation.
20. **Who Shares My Function? - Linear with Graphs and Stories** - Students will make groups by finding other students who have the same quadratic or linear function in different representations.
21. **Wind-Up Car** - Students will produce an equation from a graph, based on an engineering-context.