

Partial and Total Changes

Partial and Total Changes

Click [here](#) to download lesson.

Summary	Students learn that two partial changes are equivalent to a single total change. On the number line, this corresponds to the idea of a shortcut. Three notations are emphasized: words, number lines with hopping arrows, and numerical expressions.
Goals	<ol style="list-style-type: none">1. To move between story problems, enactments on the number line, arrow/hops notation, and number sentences.2. To understand that two additive changes are always equivalent to, and can be expressed as, a single change.3. To understand that a whole story with two additive changes can be shortened to a story with one change or with merely the end point.4. To associate unary operators (+ -) with numbers: a change of +4 is different from a change of -4.5. To lay the groundwork for vector notation.
Materials	A previously assembled "number line" (Going from -10 to +20 fixed onto a wall or hanging across the classroom), Overheads, Handouts
Keywords	Full Class Discussion Negative Numbers Number Lines

Activity Plan:

Partial Changes and Overall Change

1. Changes on an unspecified amount [Whole Class]

Hold an informal discussion (no overheads) with students about the following problem:

***Your mother gives you \$2.00 and then later gives you \$3.00.
How could you represent this with numbers?***

Some students may suggest $2+3$ or $\$2.00 + \3.00 .

Suggest adding a plus sign to the first addend: this helps characterize the 2 or $\$2.00$ as a positive change. The sentence then becomes:

$$+2 +3$$

or

$$+\$2.00 + \$3.00$$

[Technically, one could argue for $(+2) + (+3)$ but this would be overly confusing and unnecessary at this point.]

Now, a key idea:

How many times did the amount of money you had change? [twice: It changed when she gave you money the first time; it changed when she gave you money the second time.]

What if your mother wanted to give you the money at once? [She could have given \$5.00 at once, instead of in two parts, \$2.00 and \$3.00]

These two distinct conceptualizations correspond to different numerical expressions:

- Two changes in amount: $+\$2.00 + \3.00
- One change in amount: $+\$5.00$, also known as the **overall** or **total change**
- The two expressions are the same amounts: $+\$5.00 = +\$2.00 + \$3.00$

Placing $\$5.00$ on the left side is intentional; the equals sign cannot be interpreted as "yields".

Now try this one:

You earned \$10.00 and then spent \$3.00.

How could you represent this with numbers?

We have in mind something like:

- As two changes: $+\$10.00 - \3.00
- As the overall change: $+\$7.00$
- The expressions are equal: $+\$7.00 = +\$10.00 - \$3.00$

Now try the story with the order reversed:

You spent \$3.00 and then earned \$10.00

How could you represent this with numbers? ($-\$3.00 + \10.00)

If you find it useful to illustrate these events on the number line, start at some value (e.g. zero) distinct from the addends. Do not treat the first addend as a start-point! Otherwise it will not seem to be a change.

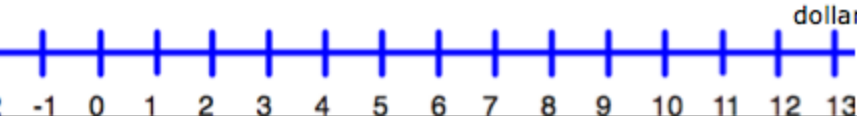
2. Multiple changes on the number line [Whole Class]

Distribute the handout on page 1.

Problem 1:

Ask a student to enact, on the number line hanging in the class, the story about Sam:

Sam had \$2.00. Last week he earned \$4.00 and then spent \$1.00.

With Words	Sam had \$2.00. Last week he earned \$4.00 and then spent \$1.00.	
On A Number Line (dollars)		
With a number sentence	$2 + 4 - 1$	The beginning amount and two changes
	$2 + 3$	Beginning and the total change
	5	The end amount

The story mentions three amounts, \$2.00, \$4.00, and \$1.00. The end value (\$5.00) can be regarded as an unknown amount. Only the beginning and ending values are associated with specific locations (and labels) on the number line. The middle two values correspond to intervals, distances, lengths on the number line. Make sure to ask:

Didn't the problem have \$4.00 in it? Where are those four dollars? (This cannot be answered by pointing to the label 4 on the number line!)

Is it plus \$4.00 or minus (take away) \$4.00?

Do the same for the \$1.00 that was spent:

Where is the \$1.00 that Sam spent?

Don't accept a point on the number line as an answer. We're talking about a distance or interval here.

Ask the students in the class to diagram the movements and locations on the number line for problem 1 in their handouts. They should label the arrow lengths¹. They should also be able to talk through the story as they move along their drawing.

Number expressions: Encourage the students to express "what happened" as $2 + 4 - 1$. Ask how many changes in amount the number expression shows (2).

Ask them how to show the two changes as one. That is,

The amount of money Sam had changed twice last week, right? Well, what was the overall change in his amount? If we compare what he had at the beginning of the week with what he had at the end of the week, what is the difference?

In order to solve the overall change, students may need to first determine Sam's amount at the end of the week (\$5.00).

Spend some time getting students to see that the three expressions are different ways to show \$5.00 (and so they are equal):

What is another name for $4-1$, the second change? (3)

What is another name for $2+3$? (5)

What is another name for $2+4-1$? (5)

¹ The label specifying the magnitude of the arrow hop conveys the direct distance from the starting to the ending point, not the length of the arc.

Problem 2:

Alice had 7 candies. She bought 5 more candies at the store. On the way home, she ate 3 candies.

With Words	Alice had 7 candies. She bought 5 more candies at the store. On the way home, she ate 3 candies.	
On A Number Line (dollars)		
With a number sentence	$7 + 5 - 3$	The beginning amount and two changes
	$7 + 2$	Beginning and the total change
	9	The end amount

This problem is similar to the first, and the two changes, $+5 - 3$, can be joined as “the total change in candies (what happened) after Alice left her home”. After she returns home, she has 2 more candies than she had when she left even though she ate some candies.

Problem 3:

Frank had 9 candies. He bought 2 more candies at the store. On the way home, he ate 6 candies.

With Words	Frank had 9 candies. He bought 2 more candies at the store. On the way home, he ate 6 candies.	
On A Number Line (dollars)		
With a number sentence	$9 + 2 - 6$	The beginning amount and two changes
	$9 - 4$	Beginning and the total change
	5	The end amount

This problem is more difficult than the last: Frank comes home with four candies less than he left with. It is difficult to construe what happened only in terms of the candies purchased. The net effect was -4 ; Frank must have gone into the candies he had to begin with! So when he returned home he actually had few candies than when he left.

3. Homework (Page 2)

It is very similar to the problem given in class.

Overhead and Handout: Showing Changes in Different Ways (Page 1)

Name: _____ Date: _____

With Words	Sam had \$2.00. Last week he earned \$4.00 and then spent \$1.00.
On A Number Line (dollars)	
With a number sentence	_____ The beginning amount and two changes
	_____ Beginning and the total change
	_____ The end amount

With Words	Alice had 7 candies. She bought 5 more candies at the store. On the way home, she ate 3 candies.
On A Number Line (dollars)	
With a number sentence	_____ The beginning amount and two changes
	_____ Beginning and the total change
	_____ The end amount

With Words	Frank had 9 candies. He bought 2 more candies at the store. On the way home, he ate 6 candies.
On A Number Line (dollars)	
With a number sentence	_____ The beginning amount and two changes
	_____ Beginning and the total change
	_____ The end amount

Overhead and Homework: Showing Changes in Different Ways (Page 2)

Name: _____ Date: _____

With Words	Ten days ago my tomato plant was 4 inches tall. A week ago it grew 3 inches. Yesterday it grew 2 inches more.	
On A Number Line (inches)		
With Numbers		The beginning amount and two changes
		Beginning and the total change
		The end amount

With Words	Frank had 9 candies. Today he bought 2 more candies for 50 cents. On the way home, he ate 6 candies.	
On A Number Line (inches)		
With Numbers		The beginning amount and two changes
		Beginning and the total change
		The end amount

Lydia keeps her money in a piggy bank. Last week she spent and earned some money. She now has **\$5.00 more** in her piggy bank than she had last week.

One possible story [see table below] is this: Maybe she spent \$1.00 and earned \$6.00
What other stories could explain the increase of \$5.00 in her piggy bank?

	First Change in Amount	Second Change in Amount	Total Change
Possible Story 1	She spent \$1.00	She earned \$6.00	+\$5.00
Possible Story 2	She spent \$3.00		+\$5.00
Possible Story 3			+\$5.00