

# Interpretation of Stories

## Interpretation of Stories

### Third Grade Lessons

1. **All Things Being Equal III** - The students will write equations to represent verbal statements and successive transformations that maintain or do not maintain the equality.
2. **Candy Boxes** - This class centers on the possible amounts of candies two children, John and Maria, have. They each have the same, unspecified number of candies inside their own candy box. John has, in addition, one extra candy and Maria has three extra candies. What are the possible total candies they might have?
3. **Comparing Different Functions** - The students will discuss, represent, and solve a verbal problem involving the choice between two functions.
4. **Comparing Graphs** - Students are given an hourly rate of pay and infer coordinates for (h, \$) over a range of hours. They produce a table and a graph of work-pay. Then they produce another graph for another rate of pay and discuss differences in time and pay.
5. **Comparison Problems & Tables** - This class will be used to review concepts and representations as applied to the solution of verbal comparison problems and to work on function tables.
6. **Comparisons and Attributes** - Work with comparisons and comparison operators ( $=$ ,  $<$ ,  $>$ ).
7. **Dinner Tables I** - Students work with a function relating number of tables to the number of available seats. One table seats 4, two tables seat 8, three tables seat 12....
8. **Dinner Tables II** - Students work with a function relating the number of tables (in a straight line) to the number of available seats. One table seats 4, two tables seat 6, three tables seat 8....
9. **Formulas and Stories** - The students will be required to work with the relation between different mathematical expressions (formulas) and stories.
10. **Functions - Earning Money** - The students will create tables and equations from given stories. The functions are additive and multiplicative.
11. **Multiple Number Lines** - Students continue to learn that two partial changes that start at different points on the number line are equivalent. At the end, they will work with notation for variables ( $N + 5 - 3$  or  $N + 2$ ).
12. **N-Number Line II** - Students use the N-Number line to make generalizations about an unknown amount of money in a piggy bank.
13. **Part-Whole Relations** - This class follows the discussion from the Candy Boxes I class. The challenge is to work with a visual representation of the relationships among the various quantities in the candy box problem and to relate the visual and numerical information contained in visual diagram(s) to verbal descriptions and to algorithms for finding unknown values.
14. **Piggy Banks** - The whole lesson revolves around a multipart story problem involving changes in two quantities over several days of a week. The initial quantities are equal yet unknown. Then transformations are applied to the quantities. Students are asked to compare the quantities throughout the week even though only their relative relationship can be determined.
15. **Rules and Formulas** - Students are given a rule and a data table supposedly generated according to the rule. Students evaluate whether: (1) the proper rule has been applied and (2) the result is correct.
16. **Three Heights** - In this class we will explore: (a) How the children deal with comparisons, (b) How they draw inferences from comparisons, and (c) How they represent comparisons between three unknown amounts.

### Fourth Grade Lessons

1. **Cartesian Candy Bars I** - We compare ratios of various ordered pairs in a Cartesian grid. The initial discussion concerns the space as a whole; the task will focus on selected points and on the ratio of the dependent variable to the independent variable.
2. **Comparing Strips of Unmeasured Lengths I** - The class is the first of a series that will focus directly upon the algebraic representation of measurements and their multiplicative relations. Children are asked to compare the lengths of strips, to describe the relationships between them in multiple ways, and to demonstrate that the relationships they represent are true.
3. **Comparing Strips of Unmeasured Lengths II** - The class is the second of the "Strips of Unmeasured Lengths" series that will focus directly upon the algebraic representation of measurements and their multiplicative relations. Children are asked to compare the lengths of strips, to use algebraic notation to describe the relationships between them, and to demonstrate that the relationships they represent are true.
4. **Comparing Strips of Unmeasured Lengths III** - This is the third lesson in the "Strips of Unmeasured Lengths" series that focuses directly upon the algebraic representation of measurements and their multiplicative relations. We will work with the relationship  $B = 3S$ , focusing on equations and their verbal descriptions and on true and false equations and statements.
5. **Comparing Functions** - This lesson is split into two days. In the first class, the students will analyze eight basic graph shapes and will represent and solve a verbal problem involving the choice between two functions. In the second one they will be asked to choose, among the eight basic graph shapes, the ones that matches specific situations.
6. **Consistency** - Children choose pairs of numbers that maintain the relationship of 1 to 3 that is given in a statement, and they explain why they believe the relationship is maintained.
7. **Equations and Inequalities** - Students will work with equations and inequalities, first with simple ones and later with comparisons of two functions. The Wallet Problem, introduced in a previous lesson, will provide the background context.
8. **Evaluation Problem** - Students will be given a problem that asks about the amount of money each person has, based on the amount in a piggy bank. They will be given one graph and asked to draw the second graph.
9. **Fourth Grade Assessment I** - This is a written assessment where children will interpret and determine the truth or falsehood of equations and statements that describe comparisons between quantities.
10. **Fourth Grade Assessment I Review** - Children discuss responses to problems where they interpret and determine the truth or falsehood of equations and of statements that describe comparisons between quantities.
11. **Fourth Grade Assessment II** - This is a written assessment where children will interpret and determine the truth or falsehood of equations and of statements that describe comparisons between quantities.
12. **Fourth Grade Assessment III** - This is a written assessment where children will be asked to interpret graphs and to interpret and determine the truth or falsehood of equations and statements that describe comparisons between quantities.
13. **Graphing A Story** - A trip is described in miles, hours, and miles/hr. Students produce a graph from the description. They then produce a table from the graph and answer questions about the trip.
14. **Graphing Halves and Doubles** - Children work on a problem about distance and time and compare two rates: half a meter per second and two meters per second.
15. **Graphing Thirds and Triples** - Children work on a problem about distance and time and compare two rates: one third of a meter per second and three meters per second.
16. **Multiplicative Candy Boxes I** - This class centers on the possible amounts of candies two children, Juan and Marcia, have. Juan has a box of candy and Marcia has twice as much candy. What are the possible amounts of candies they might have?

17. **Multiplicative Candy Boxes II** - This class is a continuation of the Multiplicative Candy Boxes I lesson. It centers on the possible amounts of candies two children, Juan and Marcia, have. Juan has a box of candy and Marcia has twice as much candy. What are the possible amounts of candies they might have?
18. **Three Heights Review** - In this class we will explore: (a) How children deal with comparisons, (b) How they draw inferences from comparisons, and (c) How they represent comparisons between three unknown amounts.
19. **Three to One** - Children discuss and produce verbal and mathematical statements on the proportion,  $S:L :: 1:3$ , that is, on the function  $f(x) = 3x$  and on its inverse  $f^{-1}(x) = 1/3x$
20. **Two Phone Plans I** - Students compare two phone plans, one of which has a lower rate, but a monthly basic charge; the other has a higher rate but no basic charge.
21. **Two Phone Plans II** - Students will work on the comparison between two phone plans (also used in the lesson "Two Phone Plans I"), one of which has a lower rate, but a monthly basic charge, the other has a higher rate but no basic charge.
22. **Varying Speed** - Children are asked to tell a story about a trip depicted through a graph that has varying slopes/speeds.
23. **Varying Velocity** - Children are asked to tell a story about a trip depicted through a graph that has varying slopes/velocities.
24. **Wallet Problem I** - Students compare the amounts of money two students have. The amounts are described relationally but not through precise dollar amounts.
25. **Wallet Problem II** - Students will be given a wallet problem. They will be asked to compare the amounts of money two students have. The amounts are described relationally but not through precise dollar amounts.
26. **Wallet Problem III** - Students will continue working with the wallet problem. They will be shown a graph for Mike's amounts and asked to (a) determine whether it represents Robin's or Mike's money and (b) to predict where the line for Mike would fall. Later they will plot Mike's amounts and will discuss why the lines cross.

#### Fifth Grade Lessons

1. **Arcade** - Students are told a story about two children, each of whom has a certain amount of money, but only one of whom has an amount known to us. After a series of events they happen to end up with the same amount of money.
2. **Basic Function Shapes** - In this lesson, the students will (a) discuss, represent, and solve a verbal problem involving the choice between two functions; (b) choose, among 8 basic graphs (7 distinct shapes), the one that matches specific situations; and (c) write stories to match a specific graph shape.
3. **Elapsed Time** - A variant of the train crash problem is used to address questions about elapsed time. The task is to determine where a train is, given a certain time.
4. **Enacting and Solving Equations** - Students enact and discuss a situation where two children have amounts of candies. Some of the candies are visible, others are inside opaque tubes or boxes. After considering multiple possibilities they are told that the children have the same amount of candies. The situation corresponds to the equation  $3x + y + 6 = x + y + 20$ , where  $x$  is the amount of candies per tube and  $y$  is the amount of candies per box. Students will be asked to discuss and to represent the situation, to solve the equation that corresponds to the situation, and to solve other written equations with similar structure.
5. **Equations and Graphs** - Students will further compare two linear functions in the context of evaluating two plans for shoveling snow. One plan has two parts: a basic charge plus a charge based on the number of square meters cleared. The other plan has no basic charge; it only charges according to the number of square meters cleared. However the per-meter charge is higher than in the other plan. Students are asked to determine the circumstances in which the bill from each plan would be the same. They then examine the graph of the two functions and discuss how equations and inequalities relate to the graph.
6. **Fifth Grade Assessment I** - This assessment will focus on writing equations to solve verbal problems and on solving equations using syntactic rules. It is intended as a diagnostic tool to assist teachers in planning future activities.
7. **Fifth Grade Assessment I Review** - This lesson will focus on reviewing the recent in-class assessment, on writing equations for word problems, and on solving equations.
8. **Fifth Grade Assessment II** - This assessment will focus on writing equations to solve verbal problems and on solving equations using the syntactic rules of algebra.
9. **Fifth Grade Assessment III** - This assessment will focus on writing equations to solve verbal problems and on solving equations using the syntactic rules of algebra.
10. **Phone Plans** - Students will compare two linear functions in the context of evaluating phone plans. One plan has two parts: a basic charge plus a charge based upon the number of minutes used. The other plan has no basic charge; it only charges according to the minutes used. However the per-minute charge is higher than in the other plan. Students are asked to determine the circumstances in which the monthly bill from each plan would be the same. They then examine the graph of the two functions and discuss how equations and inequalities relate to the graph.
11. **Review on Graphs and Equations** - In this lesson, the students will solve individually or in small groups the set of problems. For each problem, the teacher will lead a discussion based on the students' work (the teacher should identify strong and weak points in the students' work). The class is organized around four main problems. Within each problem students will answer different questions.
12. **Solving Equations II** - Students will be asked to represent and solve verbal problems requiring algebra and to use the syntactic rules of algebra to solve equations with variables on both sides of the equals sign.
13. **Solving Equations with One Variable** - Students work on a story about two children who each have a certain amount of money. The amount of one of the children is known but the other is not. After a sequence of transformations they end with the same amount of money. Students will be led to solve for the starting value by relating the equation to the events in the story. After that, they will be asked to solve another similar problem.
14. **Train Crash** - Students will compare two linear functions represented in a graph. They reason about the problem using (a) the word problem and two diagrams; (b) a graph of position vs. time; (c) a table of values (d) making expressions for each position function; and (e) solving the equation algebraically.
15. **Varying Rates of Change** - Students will compare three functions, two of which are nonlinear, that tell the story of three cousins who all save \$1,000 in one year. One saves a lot the first day and less and less each day as time goes on; one saves very little the first day and more and more each day throughout the year; the last cousin saves the same amount each day. Students will be asked to predict the shape of the graph for each function and, later, to look at and describe graphs of all three cousins' savings.
16. **Wallet Review Problem** - This activity is a review of the Wallet Problem done in fourth grade. It is intended to introduce new students to some of the concepts we have covered and to refresh the memories of our old students. Students compare the amounts of money two students have. The amounts are described relationally but not through specific dollar amounts.

#### Middle School Lessons

1. **Who Shares My Function? - Linear with All Representations** - Students will work in groups after finding other students who have the same linear function represented by a story, a table, a graph, or an equation. They will attempt to explain and discuss why the different representations refer to the same function.
2. **Who Shares My Function? - Linear with Graphs and Stories** - Students will make groups by finding other students who have the same quadratic or linear function in different representations.