Multiplicative Candy Boxes I

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Summary	This class centers on the possible amounts of candies two children, Juan and Marcia, have. Juan has a box of candy and Marcia has twice as much candy. What are the possible amounts of candies they might have?
Goals	1. Work with variables, function tables, and algebraic representation for variables.
Materials	Three Opaque Boxes that hold up to 100 candies (Plastic Card File Boxes also work) labeled with their owners (Juan, Marcia), Individually Wrapped Hard Candy (i.e.: Jolly Ranchers), Rubber Bands (to help keep the boxes closed), Handouts, Overheads
Keywords	Contextualized Situations Describing Magnitudes Full Class Discussion Function Representations Hands-On Activity Interpretation of Stories Production of Algebraic Expressions Production of Stories Production of Tables Ratios
Foci	Variables and Function tables; Use of symbolic notation for variables as standing for the possible amount inside a box and to represent Juan's and Marcia's functions; The ratio between two multiplicative functions is constant: for any possible amount in the box, the ratio of Juan's to Marcia's amount is always 1:2;

	Representing the situation in words, pictures, tables, and algebra notation, from the point of view of Juan or Marcia (Marcia always has twice as much as Juan, Juan always has half as much as Marcia).

Introduction:

1. Juan's amount is a function of the number of candies in the box. If the amount in the box is n, then his total is n. In mathematical notation, $f(n)=n$ is the function. Another way to express this is through mapping notation: $n \rightarrow n$.

Marcia's amount corresponds to the function $f(n)=2n$.

n refers to the number of candies in a box, whatever that number is. Since n stands for any possible number, we can say that Juan has n candies because that will confirm that he has as a total only what is in the box.

However, we cannot refer to Marcia's candies as n (if we are already using the n to signify the number of candies in the box.) If n were used for all three of these purposes (candy in box, Juan's candy, Marcia's candy) this would give the incorrect idea that each of the values were the same.

2. It is totally arbitrary which letter we use to represent the unknown amount in a box. But once that letter is assigned a meaning, we can't continue to use it arbitrarily.

3. Don't expect new students to spontaneously produce the idea of using a letter to represent a variable. They may guess a specific value for the amount of candies in a box. Others may wish not to make a prediction, or to use a question mark or line segment to represent the amount in the box. These cases are opportunities for bridging to the convention of using a letter for a variable. (Here, students who have previously participated in early algebra lessons should be encouraged to explain how letters may be used to represent any number.)

Activity Plan:

Comparing the Boxes

1. Comparing the Amounts (Whole Class)

Show three identical boxes of candies, where one belongs to Juan and two belong to Marcia. Explain:

Juan has one box of candies and Marcia has two boxes.

The three boxes have exactly the same number of candies in them.

What can we say about how many candies they have?

Talk about it with the class. What do they know about how many candies Juan and Marcia each have? What can they say? What do they know for certain? What can they say about the relationship between their amounts?

2. Representing the problem (Individual)

Distribute the class handout (Page 1) and display the overhead (Page 1) with the problem. Ask the children to work individually to write and/or draw what they know about the boxes in their handouts.

Collect the handouts and observe varieties of answers. Choose a few examples to be discussed. Some possible representations include:

- Iconic or numerical instantiations (drawing of boxes with the same number of dots in each, statements that Juan has 6, Marcia has 12; Juan has 0, Marcia has 0)
- Line segments representing diverse amounts
- Symbolic (Question marks used for the amount in each box or use of letter(s) to represent amounts in each box).

Put examples from children on board. Focus on:

- How children have represented differently the amounts for Juan and Marcia.
- How the ratio between Juan's amount and Marcia's amount is represented.

Show on the blackboard two vertical line segments, one of them being twice the length of the other, and ask which one might stand for Juan's amount, which for Marcia's amount.

3. A Table of Possible Values (Whole Class)

Present the table of possible values on the overhead from Page 2 to the class. Explain that their job is to tell us how many candies they think there are in each box and, for each possible amount in a box, how many candies Juan and Marcia may have.

Refer to students' suggested amounts for a box not as predictions but as possible values. We want to downplay the notion that we're trying to guess the single correct value.

Write each student's name in the first column, on the respective row. That way, you can refer to the different row's as "Paul's prediction", "Susan's prediction" etc. As "predictions" are entered make sure to note whether some students think others' "predictions" are not sensible or possible.

For each prediction, ask the students to state how many candies, in total, Juan would have, and how many candies Marcia would have. Write these amounts in the appropriate cell.

If there is an inconsistency in the number presumed to be in the box and the numbers for Juan and Maria's amounts, allow a student to change the answers. For each change, strike through the original number suggested (leaving it on the overhead) and place the new value next to it.

After the table has been tidied up, ask students to state whether they see any pattern. Carefully try to assess the pattern they are trying to articulate. Eventually, if not right away, some students will come up with the idea that Marcia will always have twice as much as Juan, and Juan will always have 1/2 as much as Marcia. Refer to this as the *ratio* in their amounts. But if the students use other expressions that are consistent with the mathematics, you can use those also.

Discuss what they see in the table. Ask them, "How could we make them have the same amount?"

Explain that since we don't know yet how many candies are in each box, we are going to say that there are *n* candies and ask:

> *If n is the number of candies in a box, what can we write about the total number of candies that Juan has? [n]*
>
> *What can we write about the total number of candies that Marcia has? [2n]*

Wait for answers, discuss them, and guide the children to propose and/or to adopt notation with a letter standing for the amount in a box.

Propose to use *s* to represent the number of candies that Marcia has.

If s is the number of candies that Marcia has, what can we write about the total number of candies that Juan has? [s/2]

What can we write about the number of candies in a box? [s/2]

4. Other contexts (Whole Class)

Ask the children:

*What if the table were about Juan's and Marcia's **heights**?*

Can you think of other stories? [Possible contexts are money, ages, number of siblings (brothers and sisters), how many miles they live from the school, someone's marbles before and after playing.]

5. Present homework (Page 3) for students (Whole Class)

Display the homework (Page 3), explain what it is about and what you expect the students to do.

Overhead and Handout: Representing Juan and Marcia's Candies (Page 1)

Name: _____ Date: _____

Juan has a box of candies.
Marcia has two boxes of candies.
The three boxes have exactly the same number of candies in them.

Draw or write something below that compares how many candies Juan and Marcia have.

Overhead and Handout: Table of Possible Values
(Page 2)

Name: _____ Date: _____

Student	Number of candies in a box	Juan's total candies	Marcia's total candies
	k		

Overhead and Homework **(Page 3)**

Name: _____ Date: _____

Tony and Joy have the same birthday (January 15).
On January 16, 2010, Tony's age is exactly twice Joy's age.

Complete the table for what could be Tony and Joy's ages

How old would Joy be if Tony is 12 years old?

How old Tony will be if Joy is 7 years old.

Joy's age	Tony's age	The ratio of their ages
7		
	12	
8		
	10	
10		
	18	
k		
	p	

Now let's use the letter h to mean: Joy's age.

If Joy is h years old, how many years old will Tony be?