Graphing Halves and Doubles

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Summary	Children work on a problem about distance and time and compare two rates: half a meter per second and two meters per second.
Goals	1. To further promote students' understanding of ratios and fractions in graphical, notational, and story contexts, with a focus on continuous quantities.
Materials	Overheads, Handouts
Keywords	Compare/Contrast Functions Contextualized Situations Coordinate Pairs Full Class Discussion Interpretation of Stories Linear Functions Production of Graphs Production of Tables Ratios Slope Small Group Work
Practical Hints	Have the students give their opinions about who walks faster. See if they can justify their views. Ask them to compare ratios and slopes. Represent in writing the ratios and the ordered pairs.
Notes about the math	There are several different ways to represent multiplication of quantities. Both methods described below are correct. But you must be careful not to be inconsistent in their use.

For the following discussion, presume that we are discussing a problem in which *Shaquille travels for **n** seconds at 2 meters per second*. The problem is to express how far Shaquille travels after a certain number of seconds. You can think of the distance as a function, say, $f(n) = 2n$.

Method 1: Showing the units

Let's assume that $n = 7s$

We adults know that the answer will be 14 meters. The question is, "How can we show this?" That is, how do we start with an input of 7 seconds and get an output of 14 meters.

7s × 2m does **not** yield 14m

Nor does 7s × 2 yield 14m

Keeping track of the units requires that we conceive of the multiplication operation as

$$\mathbf{7s \times \frac{2m}{1s}}$$

The tricky part of this expression is the intensive quantity, $\frac{2m}{1s}$, which is read as "two meters per second", "two meters for each second", or even "two meters for every one second". The same quantity is correctly written as:

$$\frac{2 \text{ meters}}{\text{second}}$$

$$2\frac{\text{meters}}{\text{second}}$$

$$2\frac{m}{s}$$

$$\frac{2 \text{ meters}}{1 \text{ second}}$$

However, regardless of format, it does **not** mean that the distance traveled was two meters and the time taken was one second, as students will often assert. The expression does **not** reveal the distance or the duration of travel. It only reveals the speed.

See (Schwartz 1976-1996) for a clarification of what this means. [Semantic aspects of quantity. Unpublished paper, Cambridge, Harvard University (73 pp.). Cambridge, MA, Harvard University, Department of Education: 40 pp.]

The $\frac{m}{s}$ part of the multiplication can be thought of as an exchange function whereby meters are exchanged for seconds.

So the multiplication

$\times \frac{\mathbf{2m}}{\mathbf{1s}}$

involves two processes: (a) it doubles the input number and (b) it exchanges meters for seconds. These need to be carefully distinguished

Method 2: Suppressing the units

Let's assume now that $n = 7$. We suppress the unit of measure [seconds] here.

Likewise, we can represent the speed as 2.

And the output of 7×2 is 14.

The notation is much simpler than the case above. But it is also more mysterious because it glosses over the fact that (7) seconds have been transformed, so to speak, into (14) meters.

In the present lesson students will be multiplying while keeping track of the units. Make sure you are comfortable with the above distinctions before teaching this lesson.

Method 3: Showing only one of the units

There is a third way of conceiving multiplication in problems involving quantities. It is a bit of a compromise between the two methods shown above: distance is explicitly tracked but time is not.

The speed, two meters per second, tells us that 2 meters are traversed each second.

The distance can be thought of as 2m times the number of

	seconds. **2m x 7** yields 14m. This representation only keeps explicit track of a single unit, meters. It is less burdensome than Method 1, but it glosses over the exchange function aspect of multiplication (Schwartz, op.cit., refers to the exchange function as the referent-transforming character of multiplication to contrast it with addition, which combines like units.)

Activity Plan:

1. Generating data on a table [Group Work]

    Display and distribute Page 1. Ask children to work in pairs and develop data tables.

    If necessary, use the tables on Page 2 to discuss children's work.

2. Plotting data [Group Work]

    Distribute the handout on Page 3. Ask students to work in pairs and draw the graphs for the data in the tables.

3. Discussing students' work and focusing on ratios and ordered pairs [Whole Class]

    Discuss the students' work and explore comparisons between the two lines at the same time and at the same distance.

    Focus on ratios and the writing of ordered pairs.

4. Homework (Pages 4 & 5)

    The homework will be similar to the classroom activity but will refer to triples and thirds.

Overhead and Handout: Presenting Data Tables

Name: _____ Date: _____

Shaquille walks for a while at 2 meters per second.

Jane walks for a while at 1 meter for every 2 seconds.

Fill in the tables showing how far Shaquille and Jane walk after 1 through 10 seconds.

Shaquille

Time (seconds)	Distance (meters)
1s	$1s \times \frac{2m}{1s} = 2m$

Jane

Time (seconds)	Distance (meters)
2s	1m
1s	
4s	
10s	
5s	
8s	
9s	
3s	
6s	
7s	

Overhead and Handout: The Data Tables (Page 2)

Name: _____ Date: _____

Shaquille walks for a while at 2 meters per second.

Jane walks for a while at 1 meter for every two seconds.

Fill in the tables showing how far Shaquille and Jane walk after 1 through 10 seconds.

Shaquille

Time (seconds)	Distance (meters)
1s	$1s \times \frac{2m}{1s} = 2m$
2s	$2s \times \frac{2m}{1s} = 4m$
3s	$3s \times \frac{2m}{1s} = 6m$
4s	$4s \times \frac{2m}{1s} = 8m$
5s	$5s \times \frac{2m}{1s} = 10m$
6s	$6s \times \frac{2m}{1s} = 12m$
7s	$7s \times \frac{2m}{1s} = 14m$
8s	$8s \times \frac{2m}{1s} = 16m$
9s	$9s \times \frac{2m}{1s} = 18m$
10s	$10s \times \frac{2m}{1s} = 20m$

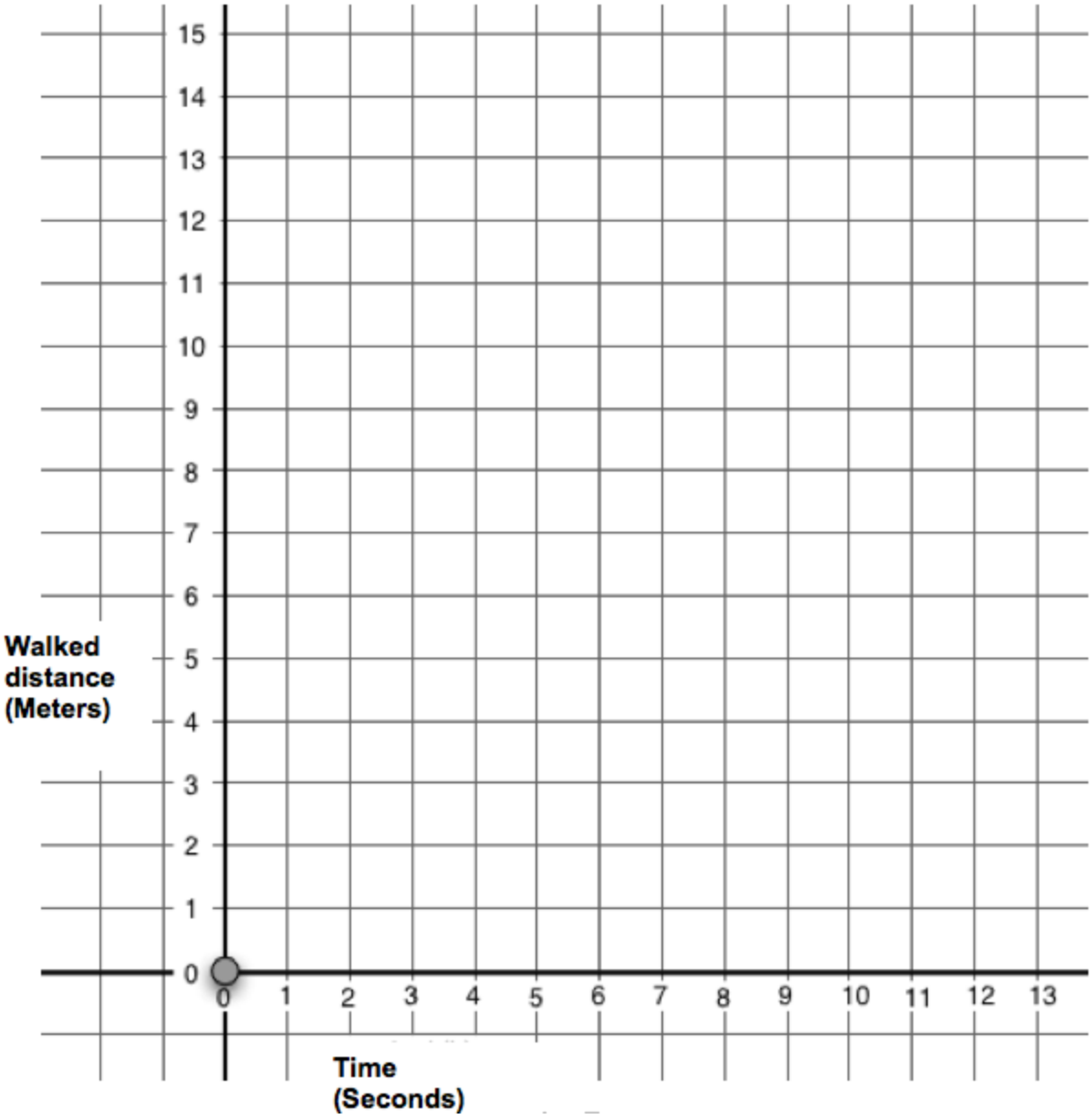
Jane

Time (seconds)	Distance (meters)
2s	$2s \times \frac{1m}{2s} = 1m$
1s	$1s \times \frac{1m}{2s} = \frac{1}{2}m$
4s	$4s \times \frac{1m}{2s} = 2\ m$
10s	$10s \times \frac{1m}{2s} = 5m$
5s	$5s \times \frac{1m}{2s} = 2\frac{1}{2}m$
8s	$8s \times \frac{1m}{2s} = 4m$
9s	$9s \times \frac{1m}{2s} = 4\frac{1}{2}m$
3s	$3s \times \frac{1m}{2s} = 1\frac{1}{2}m$
6s	$6s \times \frac{1m}{2s} = 3\ m$
7s	$7s \times \frac{1m}{2s} = 3\frac{1}{2}m$

Overhead and Handout: Graphing and Comparing Graphs **(Page 3)**

Name: _____ Date: _____

Use the data in your tables to show in the graph space below how many meters Shaquille and Jane walk at each moment from 1 through 10 seconds.



Overhead and Homework **(Page 4)**

Name: _____ Date: _____

On Sunday Shaquille walked for a while at 3 meters per second.
Jane walked for a while at 1 meter for every 3 seconds.

Make a table to show how much Shaquille and Jane walked from 1 through 10 seconds.

Shaquille			**Jane**	
Time (seconds)	**Distance (meters)**		**Time (seconds)**	**Distance (meters)**
1s	$1s \times \frac{3m}{1s} = 3m$		1s	$1s \times \frac{1m}{3s} = \frac{1}{3}m$

Overhead and Homework **(Page 5)**

Name: _____ Date: _____

Use the data in your tables to show in the graph space below how many meters Shaquille and Jane walked on Sunday from 1 through 10 seconds.